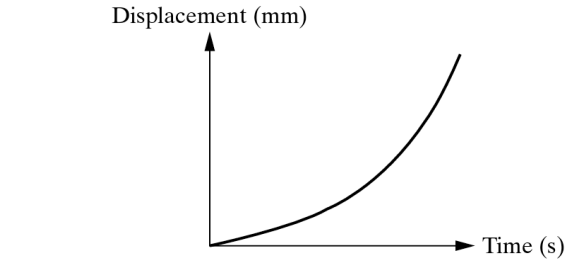
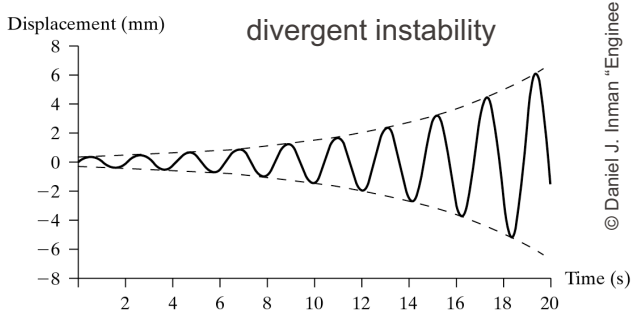
Stability - Instability

- **Stable** oscillator: limited amplitude
  - $|x| = f(t) \rightarrow$ limited
  - Mass m, stiffness k and damping coefficient $c > 0$
- **Unstable** oscillator: diverging
  - $|x| = f(t) \rightarrow \infty$
  - Stiffness k or damping coefficient $c < 0$
  - Example:
    - Inverted pendulum
      (inverted pendulum maintained by a spring)
    - Wing of a plane (flutter instability)

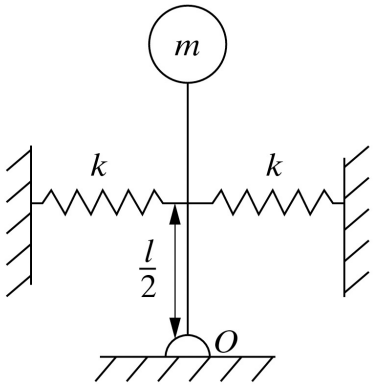


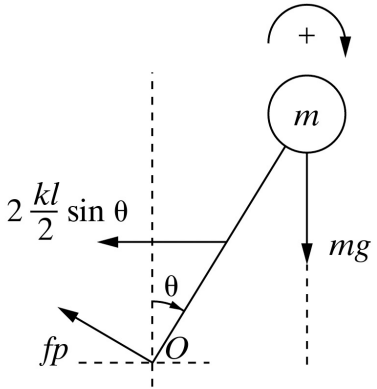
divergent instability



flutter instability

Inverted pendulum





$$ml^2\ddot{\theta} = \sum M_0$$

$$ml^2\ddot{\theta} = -\left(\frac{kl^2}{2}sin\theta\right)cos\theta + mglsin\theta$$

2x spring forces — gravitational

Daniel J. Inman "Engineering Vibration", 4th Edition, University of Michigan, 2014, Pearson, pp. 82-83

$$\Rightarrow \quad ml^2\ddot{\theta} + \left(\frac{kl^2}{2}sin\theta\right)cos\theta - mglsin\theta = 0$$

$$\theta < {}^{\pi}/_{20} \quad \Rightarrow \quad ml\ddot{\theta}(t) + (kl - 2mg)\theta(t) = 0 \qquad (cos\theta \cong 1, sin\theta \cong \theta)$$

effective stiffness

if $kl - 2mg < 0$ ⇒ **Unstable** (divergent)